647510 **BSc6MatC601x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

April/May-2022 (NEW COURSE)

Graph Theory and Complex Analysis-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1

(A) **ATTEMPT ANY TWO:** **10**

(1) Write a short note on $\boldsymbol{Königsberg\ Bridge}$ problem.

(2) Define: Simple Graph
Show that the maximum number of edges in a simple graph with $\boldsymbol{n}$ vertices is $\frac{\boldsymbol{n(n-1)}}{\boldsymbol{2}}$.

(3) Prove that a graph $\boldsymbol{G = (V, E)}$ is disconnected $\boldsymbol{iff}$ its vertex set $\boldsymbol{V}$ can be partitioned into two nonempty, disjoint subsets $\boldsymbol{V_1}$& $\boldsymbol{V_2}$ such that there exists no edge in $\boldsymbol{G}$ whose one end vertex is in subset $\boldsymbol{V_1}$ and the other in subset $\boldsymbol{V_2}$.

(B) **ATTEMPT ANY ONE :** **04**

(1) Define: (i)Subgraph (ii) Edge-disjoint Subgraphs (iii) Closed Walk (iv) Euler graph.

(2) A simple graph with $\boldsymbol{n}$ vertices and $\boldsymbol{k}$ components can have at most $\frac{\boldsymbol{(n-k)(n-k+1)}}{\boldsymbol{2}}$ edges.

Q-2

(A) **ATTEMPT ANY TWO:** **10**

(1) Define: Planar Graph
Prove that Kuratowski's first graph is non-planar.

(2) State and prove the Euler's theorem for planar graphs.

(3) Explain Decyclization by taking suitable directed graph.

(B) **ATTEMPT ANY ONE :** **04**

(1) If $\boldsymbol{G}$ be a graph with $\boldsymbol{n}$ vertices, $\boldsymbol{e}$ edges and $\boldsymbol{f}$ faces, then prove that
(i) $\boldsymbol{e \geq \frac{3}{2}f}$ (ii) $\boldsymbol{e \leq 3n - 6}$.

(2) Define: Incidence Matrix.
Prove that the rank of the incidence matrix of a connected graph $\boldsymbol{G}$ with $\boldsymbol{n}$ vertices is $\boldsymbol{n - 1}$.

Q-3

(A) **ATTEMPT ANY TWO:** **10**

(1) Discuss the mapping $\boldsymbol{w = z^2}$ in polar and Cartesian forms.

(2) Show that every linear fractional transformation except the identity transformation $\boldsymbol{w = z}$, has at most two fixed points in the extended plane.

(3) Prove that the set of all bilinear maps forms a group with respect to the binary operation of composition of mappings.

(B) ATTEMPT ANY ONE : **04**

(1) Show that under the transformation $\boldsymbol{w} = \frac{1}{z}$, the images of the lines $\boldsymbol{y = x - 1}$ and $\boldsymbol{y = 0}$ are the circle $\boldsymbol{u^2 + v^2 - u - v = 0}$ and the line $\boldsymbol{v = 0}$ respectively.

(2) Show that Bilinear mapping $\boldsymbol{w = \frac{az+b}{cz+d}}$, $\boldsymbol{z \neq \frac{-d}{c}}$, $\boldsymbol{a, b, c}$ & $\boldsymbol{d} \in \mathbb{C}$, $\boldsymbol{ad - bc \neq 0}$ is conformal.

Q-4

(A) ATTEMPT ANY TWO: **10**

(1) Define: Region & Radius of Convergence of Series.
Find region and radius of convergence for $\sum_{n=1}^{\infty} \frac{z^n}{7^n+1}$.

(2) Define: Analytic Function
State and prove Taylor's infinite series of an analytic function $\boldsymbol{f(z)}$.

(3) Expand $\boldsymbol{\frac{2z^3+1}{z^2+z}}$ in Taylor's Series at $\boldsymbol{z_0 = i}$.

(B) ATTEMPT ANY ONE : **04**

(1) Show that $\boldsymbol{\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\frac{(z-2)^n}{2^n}}$.

(2) Expand $\boldsymbol{\frac{e^z}{z(1+z^2)}}$ in Laurentz's series for $\boldsymbol{|z| < 1}$.

Q-5

(A) ATTEMPT ANY TWO: **10**

(1) If $\boldsymbol{z_0}$ is the $\boldsymbol{m^{th}(m \geq 2)}$ order pole of a complex function $\boldsymbol{f(z)}$, then prove that $\boldsymbol{Res(f(z), z_0) = \frac{\varphi^{m-1}(z_0)}{(m-1)!}}$, where $\boldsymbol{\varphi(z) = (z - z_0)^m f(z)}$ is an analytic function and $\boldsymbol{\varphi(z_0) \neq 0}$.

(2) State and prove the Cauchy's residue theorem.

(3) Evaluate: $\boldsymbol{\int_C \frac{3z^3+2}{(z-1)(z^2+9)}dz}$, where $\boldsymbol{C: |z-2| = 2}$.

(B) ATTEMPT ANY ONE : **04**

(1) Evaluate: $\boldsymbol{\int_0^{\infty} \frac{1}{(x^2+1)^2}dx}$.

(2) Evaluate: $\boldsymbol{\int_0^{\pi} \frac{d\theta}{(2+cos\theta)^2}}$.

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